Map Maker, Map Maker, Make Me a Map

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This description explains how to make a map of the locations of towns in a small region of the earth given the latitudes and longitudes of the towns.

Latitude and longitude

The earth is an ellipsoid with a polar radius of 6,378,137 meters and an equatorial radius of 6,356,752.3 meters. This deviation from a sphere of 0.3% is small enough so that we will ignore it. Thus, we can regard the earth as a sphere. To specify location on the surface of a sphere it is only necessary to give two angles — called the latitude and longitude. These are shown in Figure 1. Circles

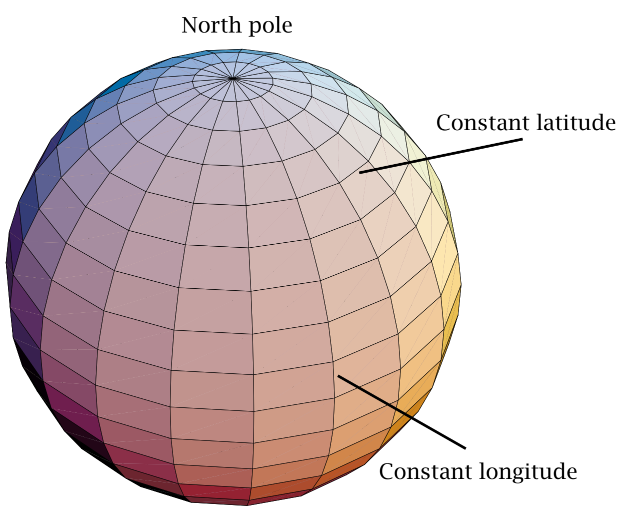

Figure 1: A spherical earth showing circles of constant longitude and latitude.

of constant longitude are great circles passing through the two poles. Note that the circumferences of these circles are the same. The reference is the great circle that passes through Greenwich, UK (called the prime meridian) which has a longitude of $0°$. The longitude has a range of $-180°$ to $+180°$ going from west to east with respect to the Prime Meridian. So if we give the longitude of a town we locate the town on a great circle with respect to the great circle that passes through Greenwich.

A plane through the earth perpendicular to an axis between the north and south poles cuts the surface into a circle of constant latitude. The circle of constant latitude at the equator is a great circle and has a reference latitude of 0°. The range of latitude is from −90° at the south pole to +90° at the north pole. Note that circle of constant latitude do not have a constant circumference and the ones at the two poles have circumferences of zero. Thus, increments of latitude do not correspond to equal distances on the surface of the earth — a degree of latitude at the equator corresponds to 111,000 meter whereas a degree of latitude at the north pole corresponds to a distance of 0 meters. Thus, maps of regions of the earth in terms of longitudes and latitudes do not correspond to maps where distance is preserved.

We see that any location on the surface of the earth can be specified uniquely by giving its longitude and latitude which gives the intersection of two circles on the surface. Both longitude and latitude are given in degrees, minutes, seconds of arc with 60 minutes in a degree and 60 seconds in a minute.

Maping from a spherical surface to a plane

The central issue in map making is how to project the surface of a sphere, or more exactly, an ellipsoid onto a plane (or other surface). There are literally 100's of ways to do this and this is a field onto itself. Fortunately, if we are concerned only with a map of a small region of the surface of the earth then we do not need to concern ourselves with the differences produced by these different methods which will be small. Thus, we can choose a simple projection scheme such as the gnomonic projection in which a plane is placed tangent to the reference location on the surface of the sphere. Each point on the sphere is projected onto the plane with a ray from the perspective point located at the center of the sphere. The geometry is shown in Figure 2. The point P is located

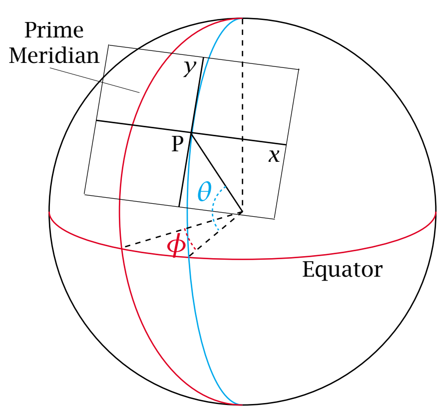

Figure 2: Geometry for projecting points on the surface of a sphere onto a plane.

on the surface of the sphere at longitude ϕ (expressed in radians) and at latitude θ (also expressed in radians). We place a plane tangent to the sphere at P with the coordinate system shown. We can locate any point on the sphere with respect to the plane with reference point P by projecting a ray from the origin of the sphere onto the plane. The coordinates of the point are given by the following formulas

$$\begin{aligned} x &= R\left(\frac{\cos\theta\sin d\phi}{\sin\theta_o\sin\theta+\cos\theta_o\cos\theta\cos d\phi}\right), \\ y &= R\left(\frac{\cos\theta_o\sin\theta-\sin\theta_o\cos\theta\cos d\phi}{\sin\theta_o\sin\theta+\cos\theta_o\cos\theta\cos d\phi}\right), \end{aligned}$$

where ϕ and θ are the longitude and latitude of the town of interest, θ_o is the latitude of the reference town (with respect to which distance is measured), $d\phi$ is the difference in longitude between the town of interest and the reference town, and r is the radius of the earth which was taken to be 3,963.1906 miles.

Map of towns in the Suchostav region

The above method was use to make the map of the Suchostav region which can be found at the URL

http://www.jewishgen.org/ShtetLinks/Suchostav/SuchostavRegion/shtetlmapmiles.html

The method was to enter the lattitudes and longitudes of all the towns in the Suchostav region into a spread sheet (Microsoft Excel). Functions were written to convert the code for the longitudes and latitudes as given in JewishGen's ShtetlSeeker and convert them to decimal degrees and then to radians. Subsequently, the coordinates were computed as indicated above with the town of Suchostav as the reference location. Distances from Suchostav were also computed as shown in Table 1.

References

Richardus, P. and Adler, R. K. (1972). *Map Projections*. American Elsevier Pub. Co., New York, NY.

Town	Latitude and longitude								
	code		decimal degrees		decimal radians				
	Lat.	Long.	Lat.	Long.	θ	ϕ	y	x	d
Berschev	4848	2603	48.8	26.05	0.8517	0.4547	-23.05	8.35	24.51
Buchach	4905	2524	49.08	25.4	0.8567	0.4433	-3.39	-21.14	21.41
Budanov	4910	2543	49.17	25.72	0.8581	0.4488	2.31	-6.78	7.17
Burakuvka	4856	2537	48.93	25.62	0.8540	0.4471	-13.82	-11.36	17.89
Chortkov	4901	2548	49.02	25.8	0.8555	0.4503	-8.07	-3.02	8.62
Dolina	4857	2543	48.95	25.72	0.8543	0.4488	-12.67	-6.81	14.39
Gorodnitsa	4911	2607	49.18	26.12	0.8584	0.4558	3.48	11.30	11.83
Grimaylov	4920	2602	49.33	26.03	0.8610	0.4544	13.84	7.51	15.75
Gusyatin	4904	2613	49.07	26.22	0.8564	0.4576	-4.57	15.86	16.51
Jagielnica	4857	2544	48.95	25.73	0.8543	0.4491	-12.68	-6.06	14.05
Kamenka	4914	2617	49.23	26.28	0.8593	0.4587	6.97	18.82	20.07
Khorostkov	4914	2555	49.23	25.92	0.8593	0.4523	6.92	2.26	7.28
Kopychintsy	4906	2556	49.1	25.93	0.8570	0.4526	-2.30	3.02	3.80
Kosov	4906	2538	49.1	25.63	0.8570	0.4474	-2.29	-10.57	10.81
Lanovtse	4851	2600	48.85	26	0.8526	0.4538	-19.59	6.07	20.51
Mikulintsy	4924	2536	49.4	25.6	0.8622	0.4468	18.47	-12.00	22.03
Ozeryany	4853	2557	48.88	25.95	0.8532	0.4529	-17.29	3.79	17.70
Palashevka	4858	2534	48.97	25.57	0.8546	0.4462	-11.50	-13.62	17.83
Pomortsy	4858	2526	48.97	25.43	0.8546	0.4439	-11.47	-19.68	22.78
Probuzhna	4902	2559	49.03	25.98	0.8558	0.4535	-6.91	5.29	8.71
Romanovka	4913	2534	49.22	25.57	0.8590	0.4462	5.79	-13.55	14.74
Satanov	4915	2616	49.25	26.27	0.8596	0.4584	8.12	18.06	19.80
Skala	4851	2612	48.85	26.2	0.8526	0.4573	-19.57	15.17	24.76
Skalat	4926	2559	49.43	25.98	0.8628	0.4535	20.76	5.25	21.41
Strusov	4920	2537	49.33	25.62	0.8610	0.4471	13.85	-11.27	17.86
Suchostav	4908	2552	49.13	25.87	0.8575	0.4515	0.00	0.00	0.00
Terebovlya	4918	2543	49.3	25.72	0.8604	0.4488	11.54	-6.77	13.37
Tolstoye/4916	4916	2605	49.27	26.08	0.8599	0.4552	9.24	9.78	13.45
Tolstoye/4850	4850	2544	48.83	25.73	0.8523	0.4491	-20.75	-6.07	21.62
Tudorov	4906	2547	49.1	25.78	0.8570	0.4500	-2.30	-3.77	4.42
Ulashkovtse	4854	2549	48.9	25.82	0.8535	0.4506	-16.14	-2.27	16.30
Vignanka	4902	2549	49.03	25.82	0.8558	0.4506	-6.92	-2.27	7.28
Yablanov	4909	2552	49.15	25.87	0.8578	0.4515	1.15	0.00	1.15

Table 1: For each town, the latitude and longitude are given in several units: in the code used by ShtetlSeeker, in decimal degrees, and in decimal radians. The angle in radians is $\pi/180$ times the angle in degrees. In the code used by ShtetlSeeker the latitude of Berschev as 4848 which means $48^\circ 48'$. The coordinates of the town are given as y and x in miles with respect to Suchostav. The distance from Suchostav is indicated by d.